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# THE RANDOM INCIDENCE SOUND ABSORPTION OF A RECTANGULAR ARRAY OF RECTANGULAR PATCHES OF A MATERIAL

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The random incidence sound absorption coefficient of a finite lateral extent of a highly sound absorbing material on a low sound absorbing planar surface is greater than the predicted sound absorption coefficient of an infinite lateral extent of the same material. This occurs because of the edge effect which creates a higher sound pressure on the surface of the material near its edges due to diffraction caused by the sudden change of the imaginary part of the admittance of the surface at the edges of the material. For a highly sound absorbing material whose opposite edges are separated by more than one wavelength, the edge effect is equivalent, on average across different materials, to the effective area of the material being increased by the perimeter of the material multiplied by one quarter of the wavelength of sound in air. Thus, in most cases, the random incidence sound absorption coefficient of a highly sound absorbing material varies linearly with the relative edge length of the material, which is equal to the perimeter of the material divided its area. Hence, the random incidence sound absorption of a given area of highly sound absorbing material can be increased by splitting the material into smaller pieces and separating these smaller pieces with gaps. This paper presents formulae for predicting the random incidence sound absorption of a rectangular array of rectangular patches of a porous sound absorbing material using measurements on a monolithic sample the same material. These formulae consider the interaction between the edges of the same rectangular patch and the interaction between the edges of different rectangular patches. The formulae use the measured random incidence sound absorption coefficient of one rectangular area of the material, the dimensions of the rectangle of the measured material and of the array of patches, and the wavelength.

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## 1. Introduction

In 1944, Morse and Bolt [1] defined the effective area of a patch of planar sound absorbing material mounted on a planar surface as the total sound absorption of the patch divided by the random incidence

sound absorption coefficient of an infinite lateral extent size planar specimen of the material. For the case of normal sound incidence, their Eq. (8.22) shows that this effective area is equal to the area of the patch plus the length of the free boundary of the patch multiplied by one quarter of the wavelength of the sound multiplied by the difference between the normalized specific susceptance of the surface of the patch and the normalized specific susceptance of the surrounding planar surface. The normalized specific susceptance is the imaginary part of the normalized specific admittance. The normalized specific admittance is the inverse of the normalized specific acoustic impedance which is the specific acoustic impedance normalized by dividing it by the characteristic impedance of the acoustic medium which is usually air. Morse and Bolt say that their equation “is derived for normal incidence; the correction for oblique waves will differ somewhat from this.”

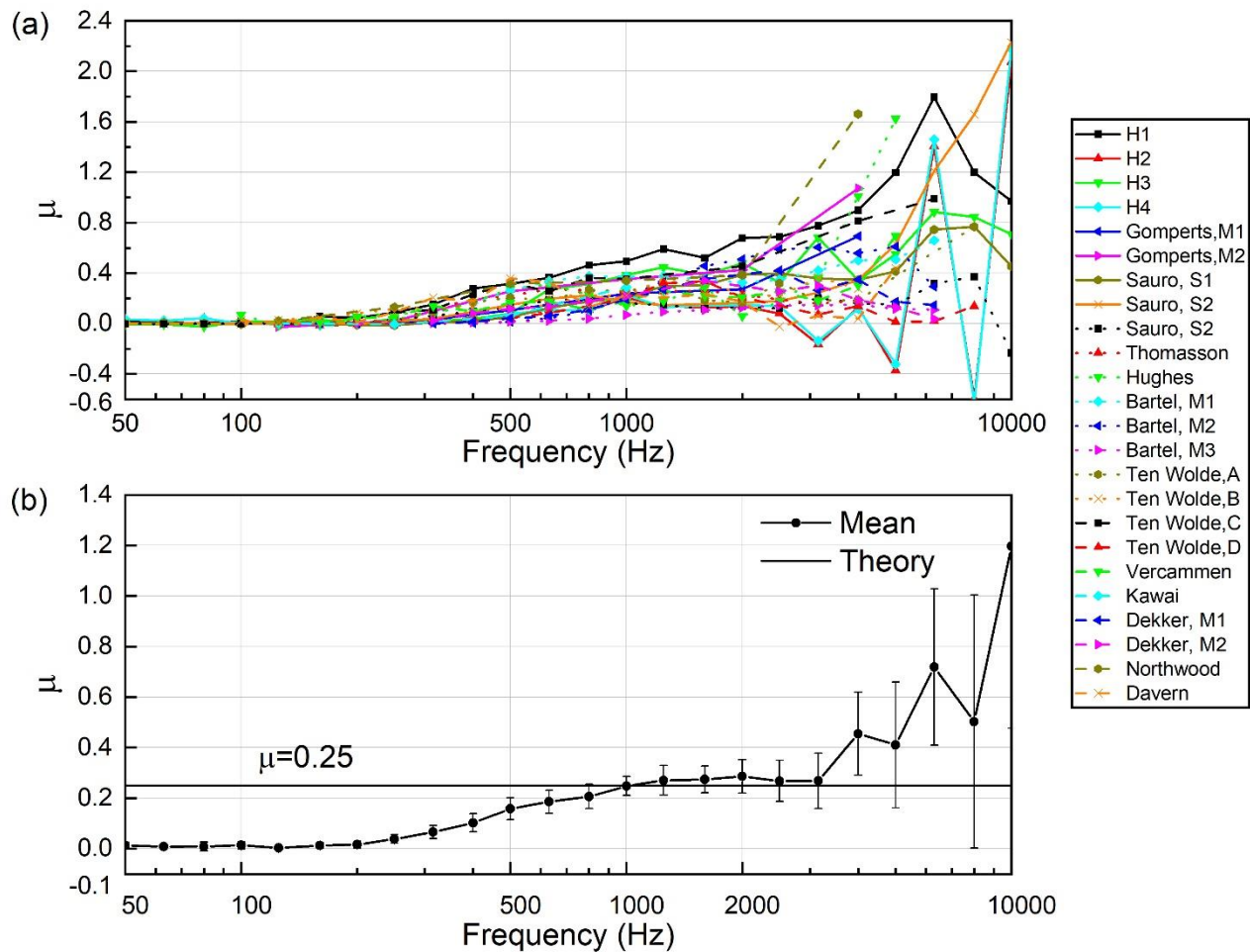


Figure 1. (a). The measured edge effect parameter  $\mu$  as a function of frequency. (b). The mean of all  $\mu$  as function of frequency compared with  $\mu = 0.25$ . The error bars represent the 95% confidence intervals.

Morse and Bolt's derivation is for the boundary between a half infinite plane of the patch material and a half infinite plane of the material surrounding the patch. They specify that the free boundary of the patch consists of only those parts of the boundary of the patch which are at least half a wavelength from surrounding walls because of the interaction of the patch with the walls. For material patches whose linear dimensions are comparable to or smaller than the wavelength of the sound there will also be interactions between the opposite parts of the boundary of the patch. If there are multiple patches of sound

absorbing material, there will be interactions between the effects of those patches which are closer than approximately one wavelength apart.

Another conclusion from Morse and Bolt's Eq. (8.22) is that the factor, by which the product of the length of the free boundary with the wavelength of the sound is multiplied to obtain the difference between the effective area and the actual area of the patch, varies with the patch material, with the patch material thickness, and with the frequency. Before and since the publication of Morse and Bolt's Eq. (8.22), there have been many measurements of this factor, or from which this factor could be derived, for random sound incidence in a reverberation room. Zhao et al. [2] gathered together 24 sets of these measurements of this factor which they denoted by  $\mu$ . Figure 1 (Zhao et al.'s [2] Fig. 3) shows that there is indeed a large variation in the experimentally determined value of  $\mu$  as predicted by Morse and Bolt's Eq. (8.22). Figure 1 also shows that the mean value of  $\mu$  is close to zero at low frequencies and is in the region of 0.25 in the mid frequency range. The mean value of  $\mu$  increases above 0.25 in the high frequency range, but the measurement certainty becomes very large because the ratio of the difference between the effective area and the actual area of the patch to the area of the patch becomes very small due to the wavelength of the sound becoming very small. This means that  $\mu = 0.25$  is a reasonable practical approximation in the mid and high frequency regions and is also used in this paper at low frequencies with the following two corrections.

In the low frequency region, there are two issues. The first is the small size of the linear dimensions of some sound absorbing patches compared to the wavelength of sound at some frequencies. In 1967, Esche [3] showed experimentally that the effective area of a patch does not increase further when the product of one quarter of the wavelength with the length of the free boundary of the patch divided by the actual area of the patch is greater than one. This means that when the patch is a very long strip, the absorption does not increase further when the width of the strip is less than half the wavelength. This is consistent Morse and Bolt's requirement that the free boundary of the patch be at least half a wavelength from the edge of the surrounding surface. When the patch is a square, its absorption does not increase further when the length of the sides of the square are less than the wavelength, which is double the value for a long strip. Esche says that this is because the mutual influence between opposite absorber edges occurs in two directions. Since this paper is assuming that  $\mu = 0.25$ , Esche's experimental result means that the effective area a patch cannot be greater than twice its actual area.

The second issue, which is more likely to occur in the low frequency range although it can also occur at any frequency, is a low sound absorption coefficient because the behaviour described above is only accurate for sound absorption coefficients that are in the region of one. Zhao et al. [2] overcame this problem by calculating an intermediate value of the sound absorption coefficient. First a corrected sound absorption coefficient which is the maximum of zero and the measured sound absorption coefficient is calculated to guard against negative measured sound absorption coefficients due to experimental uncertainty. This corrected sound absorption coefficient is multiplied by the area of the measured patch to obtain the corrected measured sound absorption of the measured patch. This corrected measured sound absorption is divided by the effective area of the measured patch to obtain the intermediate sound absorption coefficient which is greater than or equal to zero and theoretically less than or equal to one.

The differences between the effective area and the actual area of both the measured patch and the patch whose sound absorption coefficient is being predicted are multiplied by the intermediate sound absorption coefficient to obtain the modified differences. These modified differences are added to their corresponding actual areas to obtain the modified effective areas which are then divided by their corresponding actual areas to obtain the modified ratios of the modified effective areas divided by their corresponding actual areas. The predicted sound absorption coefficient is then calculated by dividing the measured sound absorption coefficient by the modified measured ratio and multiplying the result by the

modified predicted ratio. This method is given by Eqs. (19) to (22) of Zhao et al. [2] and is referred to as the linear interpolation method.

The use of  $\mu = 0.25$  for patches with high sound absorption coefficients in the mid and high frequency ranges is supported by Table 2 of Zhao et al. [4]. This table shows that the value of  $\mu$  for a patch which has the highest possible random incidence sound absorption coefficient for its shape and size is in the range from 0.251 to 0.267 when the product of the wave number with half the side length of the equivalent square is greater than or equal to the square root of two. The values in the table were calculated for a square patch and for a rectangular patch with sides whose lengths were in the ratio of 1:4 using the radiation efficiency of the patch.

Another possible approach for calculating  $\mu$  is the use of an empirical equation which is a function of frequency, thickness and density [2]. It is also possible to measure two [3] or more [5] different size and/or shape patches of the material. Surprisingly, this approach did not work quite as well as the method described above when it was tested by the authors. Daniel [6] fitted a curve to 8 values obtained from measurements on three different materials at two or three different frequencies and showed that  $\mu$  was equal to twice the modulus squared of the normalized normal incidence specific admittance. Unfortunately, Esche's [3] more extensive measurement results were significantly below Daniel's curve and were not proportional to the modulus squared of the normalized normal incidence specific admittance. However, Esche [3] was able to show that the product of  $\mu$  with the sound absorption coefficient of an infinite lateral extent size planar specimen of the material was equal the normal incidence normalized specific conductance raised to the power of 0.75 and then multiplied by 0.4. Unfortunately, this relationship only applies below the characteristic absorption frequency because above this frequency, the patch's impedance depends on the angle of sound incidence and Esche had only measured the normal incidence specific conductance. Esche defined the characteristic absorption angular frequency as the product of the resistivity and the porosity divided by the product of the density of air with the structural factor of the absorber. The structural factor of the absorber is the ratio of the total pore volume divided by the effective pore volume. The effective volume does not include dead end passages or volumes which are not connected by passages to both sides of the absorber.

## 2. Multiple patches

All four samples were composed of 0.61 m by 0.61 m square patches of material. Three of the samples consisted of the same type of glass wool (GW) material with a thickness of 51 mm and a density of 103 kg/m<sup>3</sup>. These three samples consisted of two square arrangements of 4 by 4 and 3 by 3 patches and one rectangular array of 5 by 6 patches. The fourth sample consisted of a 16 mm thick mineral wool (MW) material with a density of 194 kg/m<sup>3</sup> whose patches were arranged in a 4 by 4 square array. The 3 by 3 and 4 by 4 glass wool samples and the 4 by 4 mineral sample wool sample were measured at Armstrong World Industries, Inc.. The 5 by 6 glass wool sample was measured in the NWAA laboratory.

Esche [3] showed that the sound absorption of multiple patches was reduced if they were closer together than three quarters of the wavelength. The version of Esche's approach used in this paper is to calculate each edge's contribution to the effective area of its patch. If the width of the gap to the next patch is less than three quarters of the wavelength, multiply the edge's contribution by the width of the gap divided by three quarters of the wavelength.

Hallman has proposed the following calculation method. Integrate, normal to the edge whose contribution is being calculated, the triangular function which is equal to one half on the edge, and which linearly decreases to zero at one wavelength normal to the edge in both directions normal to the edge. Do not include the contributions to the integral for those parts of the triangular function which are over a patch. If the width of the gap adjacent to the edge and the width of the patch normal to the edge are

both greater than one wavelength, the integral is one quarter of the wavelength and the area contributed by the edge is one quarter of the wavelength multiplied by the length of the edge. If the patch width and the gap width normal to the edge are both small compared to the wavelength, the integral can have contributions from multiple gaps.

Davy proposed a variant of Hallman's method and integrated one period of a sawtooth function which was one half at the edge, and which decreased linearly to zero at one wavelength from the edge in the direction normal to the edge and away from the patch. Davy's integral only includes contributions for those parts of the sawtooth function which were over the half width of the gap adjacent to the edge. This assumed that only the gap adjacent to the edge would contribute to the area contributed by the edge and that the gap's contribution would be shared between the edges on both sides of it.

All three of these methods reduce the effective area of each patch, if necessary, so that it is not greater than twice the actual area of the patch and all three methods use the linear interpolation method described above. All three of these methods produce the same result if there is only one patch. Several variants of the three methods were tried during the research whose results are described in this paper. It became apparent that the linear interpolation method described above is always needed. Hallman proposed replacing the length of each edge in the calculation of the edge's contribution to the effective area by the maximum of half the edge length and the edge length minus the wavelength. This did not work very well.

Table 1. The Root Mean Square Error (RMSE) of the predicted random incidence sound absorption coefficient of four samples of square patches of sound absorbing material arranged in a regular rectangular array with different gaps between the patches using reverberation room measurements of the random incidence sound absorption coefficient of the zero-gap case measured in the same reverberation room.

Samples	Gap (mm)	Davy	Esche	Hallman	None
GW 4x4 16 0.61x0.61m 51 mm thick 103 kg/m <sup>3</sup>	25	0.0419	0.0426	0.0459	0.0506
	51	0.0357	0.0422	0.0600	0.0581
	102	0.0468	0.0596	0.0854	0.0926
	203	0.0588	0.0791	0.0971	0.1542
<b>Sample RMSE</b>		<b>0.0463</b>	<b>0.0579</b>	<b>0.0749</b>	<b>0.0978</b>
GW 3x3 9 0.61x0.61m 51 mm thick 103 kg/m <sup>3</sup>	305	0.0855	0.1078	0.1163	0.1860
	457	0.1025	0.1250	0.1240	0.2200
	610	0.1132	0.1188	0.1188	0.2593
	914	0.0688	0.0685	0.0685	0.2820
<b>Sample RMSE</b>		<b>0.0940</b>	<b>0.1073</b>	<b>0.1092</b>	<b>0.2397</b>
GW 5x6 30 0.61x0.61m 51 mm thick 103 kg/m <sup>3</sup>	152	0.0544	0.0531	0.0770	0.1270
	305	0.0940	0.0697	0.0735	0.2331
	457	0.1201	0.0908	0.0914	0.3019
	610	0.1141	0.1022	0.1022	0.3234
<b>Sample RMSE</b>		<b>0.0990</b>	<b>0.0812</b>	<b>0.0868</b>	<b>0.2579</b>
MW 4x4 16 0.61x0.61m 16 mm thick 194 kg/m <sup>3</sup>	25	0.0430	0.0414	0.0418	0.0488
	51	0.0562	0.0551	0.0580	0.0663
	102	0.0604	0.0625	0.0668	0.0803
	203	0.0485	0.0617	0.0666	0.0812
<b>Sample RMSE</b>		<b>0.0525</b>	<b>0.0558</b>	<b>0.0592</b>	<b>0.0704</b>
<b>Grand RMSE</b>		<b>0.0767</b>	<b>0.0784</b>	<b>0.0845</b>	<b>0.1861</b>



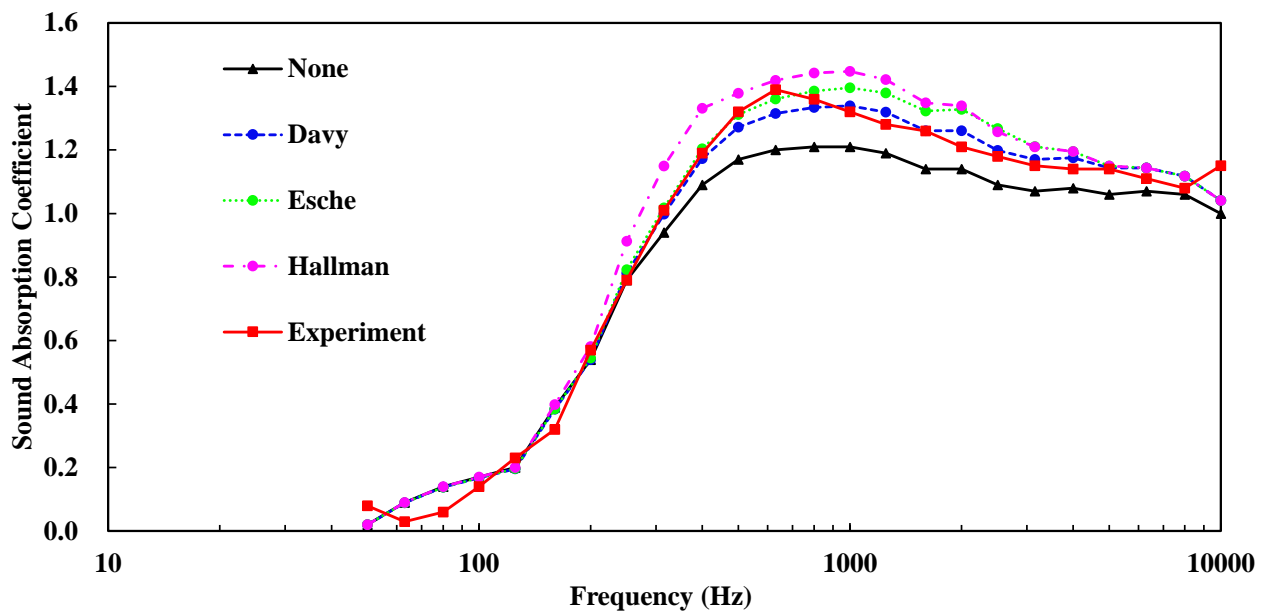


Figure 2. The predicted and the measured random incidence sound absorption coefficients of a square 4 by 4 array of 16 square 0.61 m by 0.61 m glass wool patches with a thickness of 51 mm, a density of  $103 \text{ kg/m}^3$ , and a gap of 102 mm between them measured at Armstrong World Industries, Inc. as a function of frequency.

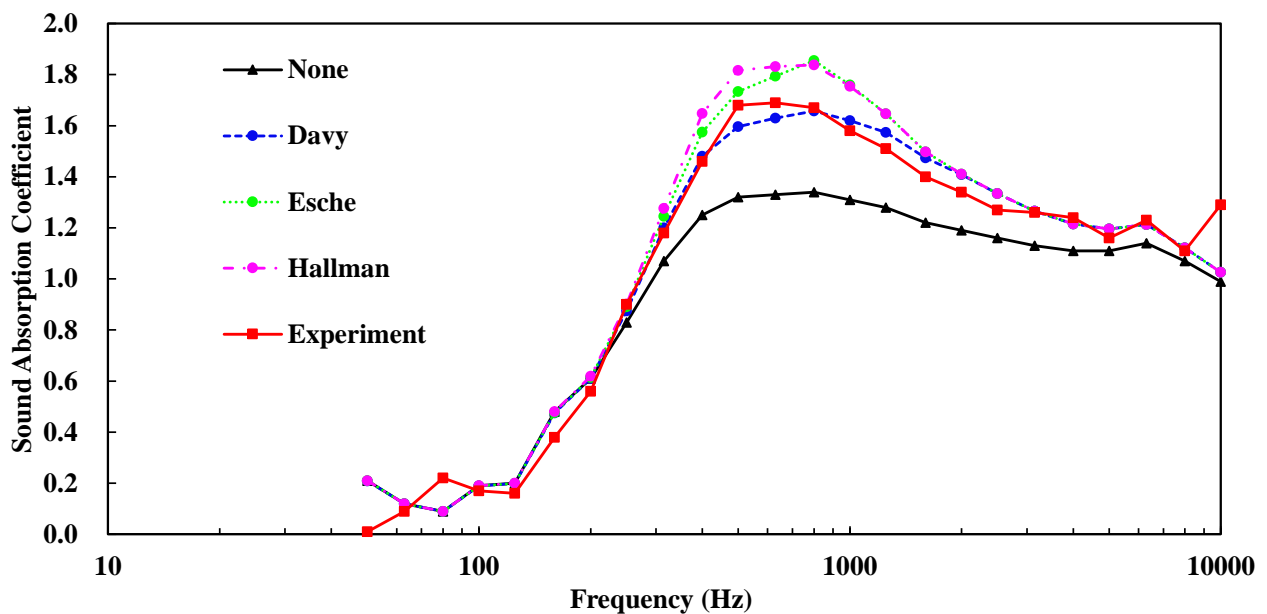


Figure 3. The predicted and the random incidence measured sound absorption coefficients of a square 3 by 3 array of 9 square 0.61 m by 0.61 m glass wool patches with a thickness of 51 mm, a density of  $103 \text{ kg/m}^3$ , and a gap of 305 mm between them measured at Armstrong World Industries, Inc. as a function of frequency.

### 3. Comparison of Predicted Values with Experimental Data

Table 1 shows the Root Mean Square Error (RMSE) of the predicted random incidence sound absorption coefficient of four samples of square patches of sound absorbing material arranged in a regular

rectangular array with different gaps between the patches. The RMSE is taken over the third octave band centre frequencies from 50 Hz to 10 kHz for each gap width and then a Sample RMSE over all gap widths is taken for each sample. A Grand RMSE is then taken across the four samples. These calculations are conducted for the three methods described in the previous section and for the case (None) when no corrections have been made to the measured values of the sound absorption coefficient.

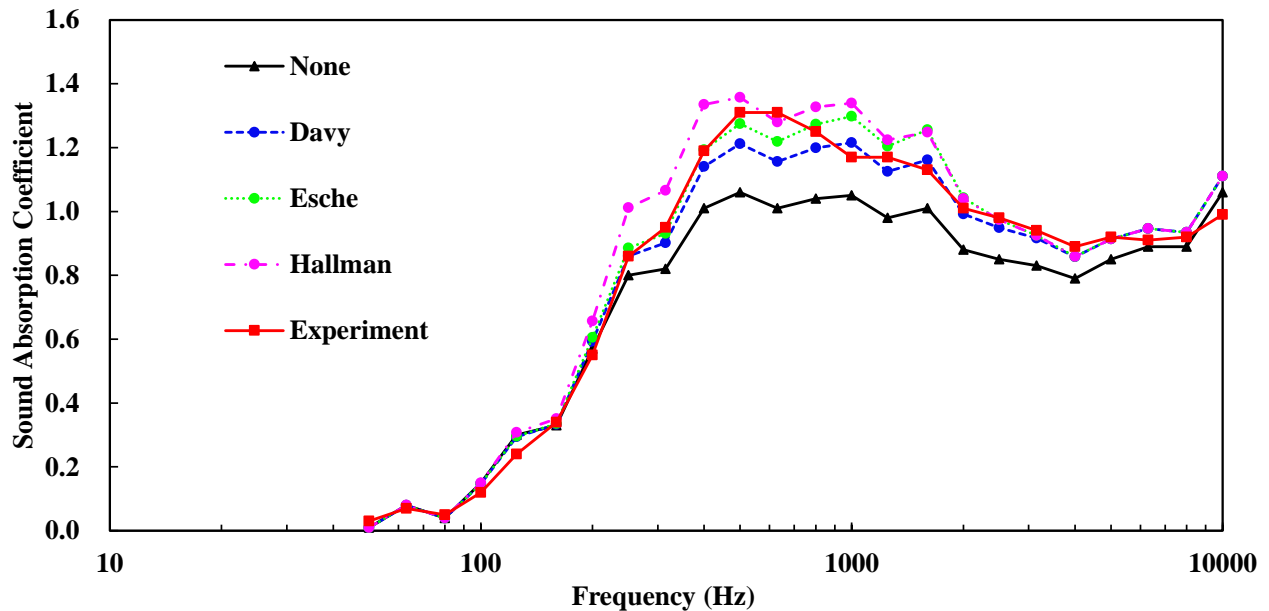


Figure 4. The predicted and the measured random incidence sound absorption coefficients of a rectangular 5 by 6 array of 30 square 0.61 m by 0.61 m glass wool patches with a thickness of 51 mm, a density of  $103 \text{ kg/m}^3$ , and a gap of 152 mm between them measured in the NWAA Laboratory as a function of frequency.

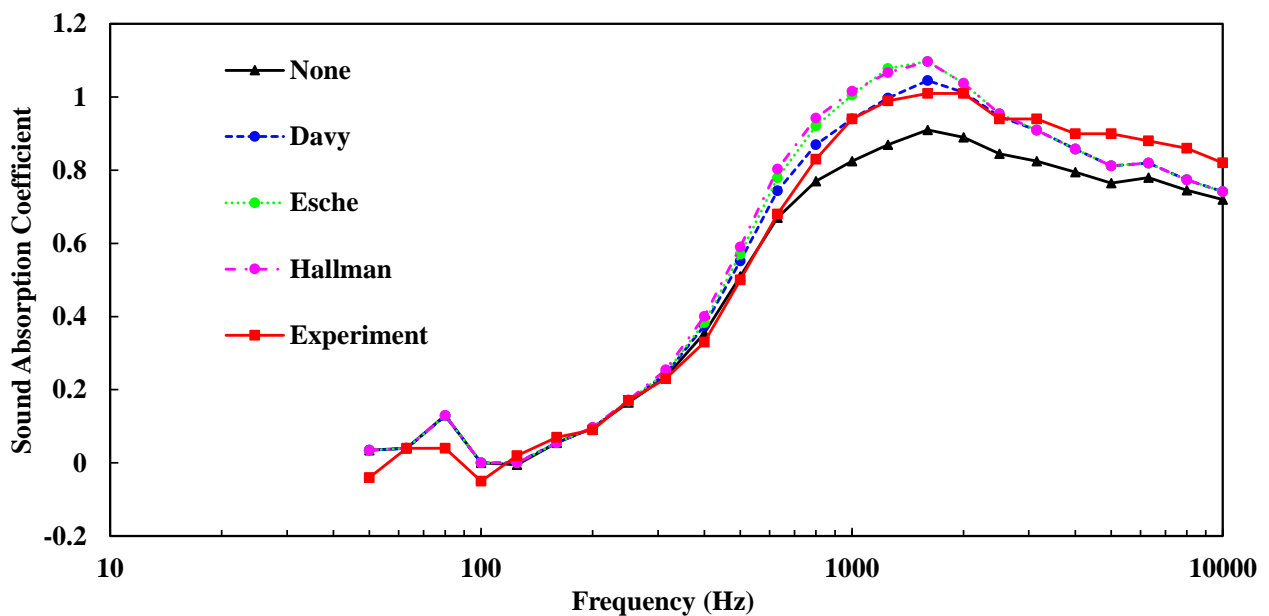


Figure 5. The predicted and the measured random incidence sound absorption coefficients of a square 4 by 4 array of 16 square 0.61 m by 0.61 m mineral wool patches with a thickness of 16 mm, a density of  $194 \text{ kg/m}^3$ , and a gap of 203 mm between them measured at Armstrong World Industries, Inc. as a function of frequency.

The zero-gap random incidence sound absorption coefficient measured values for the same number and arrangement of the patches measured in the same laboratory were used to calculate the predicted values.

The three methods all performed well. Only one out of 48 individual RMSE values were greater than the corresponding no correction individual RMSE values. All the Sample RMSE were less than the corresponding no correction Sample RMSE values. All the Grand RMSE values were less than the no correction Grand RMSE value.

Figures 2 to 5 show the predicted and the measured random incidence sound absorption coefficients of three square or rectangular arrays of 0.61 m by 0.61 m square patches of glass wool with a thickness of 51 mm and a density of 103 kg/m<sup>3</sup> and one square array of 0.61 m by 0.61 m square patches of mineral wool with a thickness of 16 mm and a density of 194 kg/m<sup>3</sup> as a function of frequency. The gap widths were 102, 152, 203 or 305 mm. The predictions are all in reasonable agreement with the measured results when allowance is made for the measurement uncertainties when measuring both the monolithic samples whose results were used in the prediction methods and the samples with gaps whose sound absorption coefficients were compared with the predictions.

## 4. Conclusion

The mid frequency, and to a lesser extent the high frequency, effective random incidence sound absorption coefficient of a sample of highly sound absorbing material can be increased by splitting the material into smaller patches and separating the patches. This paper has given three slightly different methods for predicting the effective random incidence sound absorption coefficient of rectangular arrays of rectangular patches of sound absorbing material, separated by gaps, from a measurement of the random incidence sound absorption coefficient of the sample with no gaps between its patches.

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